

General Certificate of Education  
January 2007  
Advanced Level Examination



**MATHEMATICS**  
**Unit Mechanics 2B**

**MM2B**

Tuesday 16 January 2007 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take  $g = 9.8 \text{ m s}^{-2}$ , unless stated otherwise.

**Information**

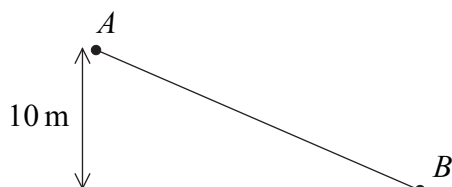
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 A child, of mass 35 kg, slides down a slide in a water park. The child, starting from rest, slides from the point  $A$  to the point  $B$ , which is 10 metres vertically below the level of  $A$ , as shown in the diagram.



- (a) In a simple model, all resistance forces are ignored.

Use an energy method to find the speed of the child at  $B$ . (3 marks)

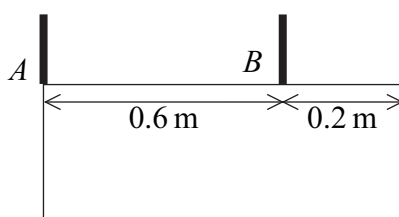
- (b) State one resistance force that has been ignored in answering part (a). (1 mark)

- (c) In fact, when the child slides down the slide, she reaches  $B$  with a speed of  $12 \text{ m s}^{-1}$ .

Given that the slide is 20 metres long and the sum of the resistance forces has a constant magnitude of  $F$  newtons, use an energy method to find the value of  $F$ .

(4 marks)

- 2 A hotel sign consists of a uniform rectangular lamina of weight  $W$ . The sign is suspended in equilibrium in a vertical plane by two vertical light chains attached to the sign at the points  $A$  and  $B$ , as shown in the diagram. The edge containing  $A$  and  $B$  is horizontal.



The tensions in the chains attached at  $A$  and  $B$  are  $T_A$  and  $T_B$  respectively.

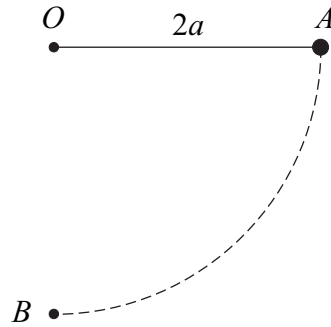
- (a) Draw a diagram to show the forces acting on the sign. (1 mark)

- (b) Find  $T_A$  and  $T_B$  in terms of  $W$ . (4 marks)

- (c) Explain how you have used the fact that the lamina is uniform in answering part (b).

(1 mark)

- 3 A light inextensible string has length  $2a$ . One end of the string is attached to a fixed point  $O$  and a particle of mass  $m$  is attached to the other end. Initially, the particle is held at the point  $A$  with the string taut and horizontal. The particle is then released from rest and moves in a circular path. Subsequently, it passes through the point  $B$ , which is directly below  $O$ . The points  $O$ ,  $A$  and  $B$  are as shown in the diagram.

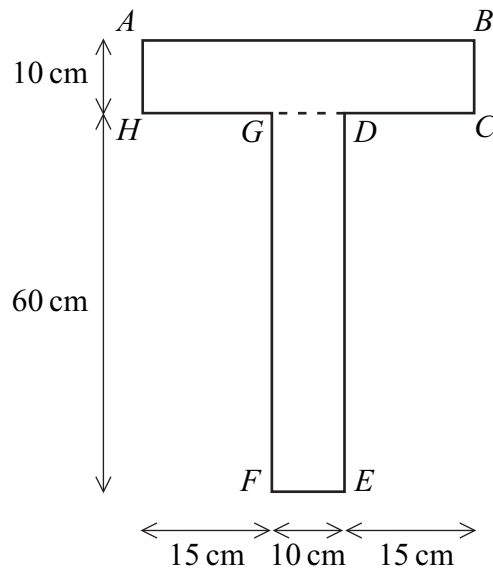


- (a) Show that the speed of the particle at  $B$  is  $2\sqrt{ag}$ . (3 marks)
- (b) Find the tension in the string as the particle passes through  $B$ . Give your answer in terms of  $m$  and  $g$ . (3 marks)

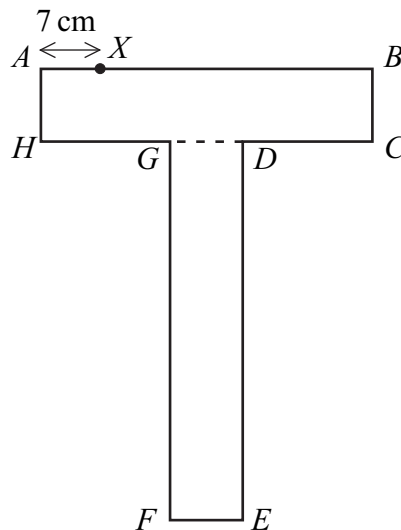
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- 4 A uniform T-shaped lamina is formed by rigidly joining two rectangles  $ABCH$  and  $DEFG$ , as shown in the diagram.



- (a) Show that the centre of mass of the lamina is 26 cm from the edge  $AB$ . (4 marks)
- (b) Explain why the centre of mass of the lamina is 5 cm from the edge  $GF$ . (1 mark)
- (c) The point  $X$  is on the edge  $AB$  and is 7 cm from  $A$ , as shown in the diagram below.



The lamina is freely suspended from  $X$  and hangs in equilibrium.

Find the angle between the edge  $AB$  and the vertical, giving your answer to the nearest degree. (4 marks)

5 Tom is on a fairground ride.

Tom's position vector,  $\mathbf{r}$  metres, at time  $t$  seconds is given by

$$\mathbf{r} = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + (10 - 0.4t) \mathbf{k}$$

The perpendicular unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are in the horizontal plane and the unit vector  $\mathbf{k}$  is directed vertically upwards.

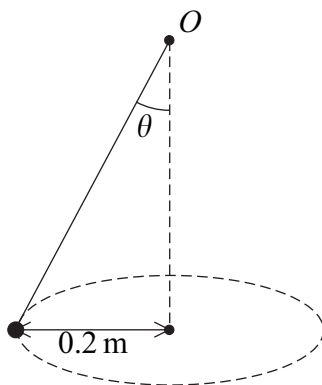
- (a) (i) Find Tom's position vector when  $t = 0$ . *(1 mark)*
- (ii) Find Tom's position vector when  $t = 2\pi$ . *(1 mark)*
- (iii) Write down the first **two** values of  $t$  for which Tom is directly below his starting point. *(2 marks)*
- (b) Find an expression for Tom's velocity at time  $t$ . *(3 marks)*
- (c) Tom has mass 25 kg.

Show that the resultant force acting on Tom during the motion has constant magnitude.  
State the magnitude of the resultant force. *(5 marks)*

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- 6 A particle is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point  $O$ . The particle is set into motion, so that it describes a horizontal circle whose centre is vertically below  $O$ . The angle between the string and the vertical is  $\theta$ , as shown in the diagram.



- (a) The particle completes 40 revolutions every minute.

Show that the angular speed of the particle is  $\frac{4\pi}{3}$  radians per second. (2 marks)

- (b) The radius of the circle is 0.2 metres.

Find, in terms of  $\pi$ , the magnitude of the acceleration of the particle. (2 marks)

- (c) The mass of the particle is  $m$  kg and the tension in the string is  $T$  newtons.

(i) Draw a diagram showing the forces acting on the particle. (1 mark)

(ii) Explain why  $T \cos \theta = mg$ . (1 mark)

(iii) Find the value of  $\theta$ , giving your answer to the nearest degree. (5 marks)

7 A motorcycle has a maximum power of 72 kilowatts. The motorcycle and its rider are travelling along a straight horizontal road. When they are moving at a speed of  $V \text{ m s}^{-1}$ , they experience a total resistance force of magnitude  $kV$  newtons, where  $k$  is a constant.

(a) The maximum speed of the motorcycle and its rider is  $60 \text{ m s}^{-1}$ .

Show that  $k = 20$ .

(3 marks)

(b) When the motorcycle is travelling at  $20 \text{ m s}^{-1}$ , the rider allows the motorcycle to freewheel so that the only horizontal force acting is the resistance force. When the motorcycle has been freewheeling for  $t$  seconds, its speed is  $v \text{ m s}^{-1}$  and the magnitude of the resistance force is  $20v$  newtons.

The mass of the motorcycle and its rider is 500 kg.

(i) Show that  $\frac{dv}{dt} = -\frac{v}{25}$ .

(2 marks)

(ii) Hence find the time that it takes for the speed of the motorcycle to reduce from  $20 \text{ m s}^{-1}$  to  $10 \text{ m s}^{-1}$ .

(6 marks)

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